Exploration of graphs with excluded minors

Júlia Baligács, TU Darmstadt



joint work with Yann Disser, Irene Heinrich, Pascal Schweitzer

The online graph exploration problem

Example: visit all cities in an unknown area

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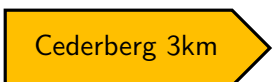
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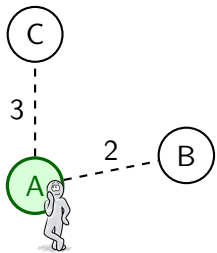
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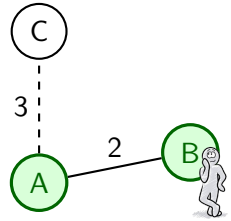
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Cederberg 4km

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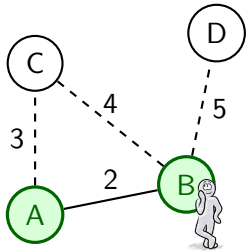
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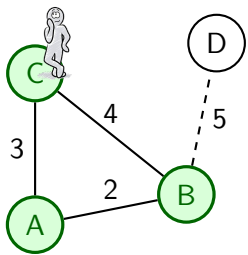
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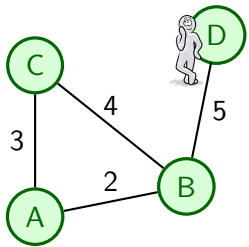
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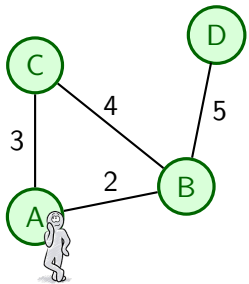
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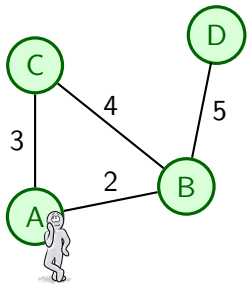
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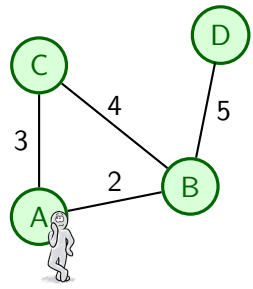
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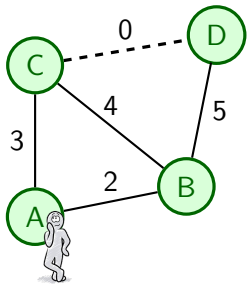
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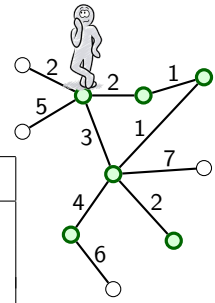
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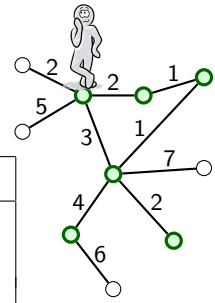
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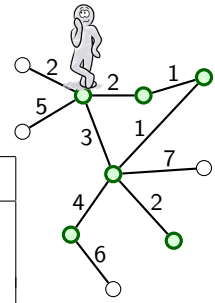
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Open question: (Kalyanasundaram and Pruhs, 1994)
Is there a constant-competitive algorithm for online graph exploration?

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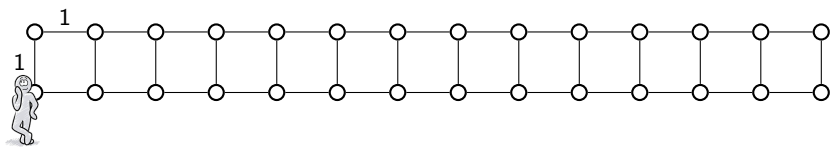
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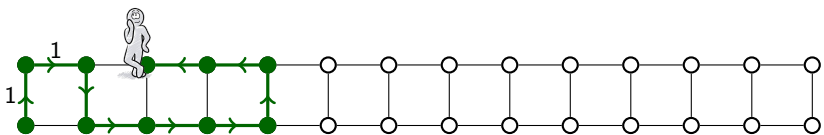
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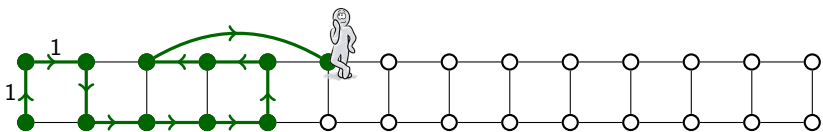
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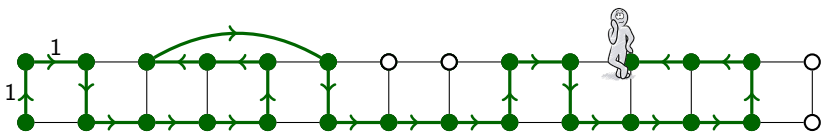
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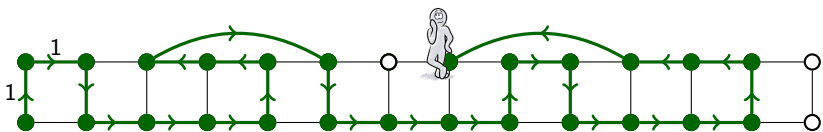
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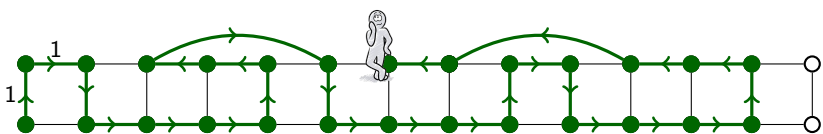
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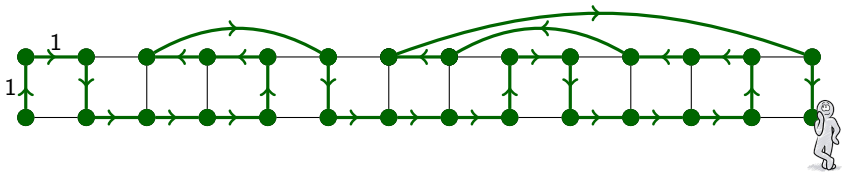
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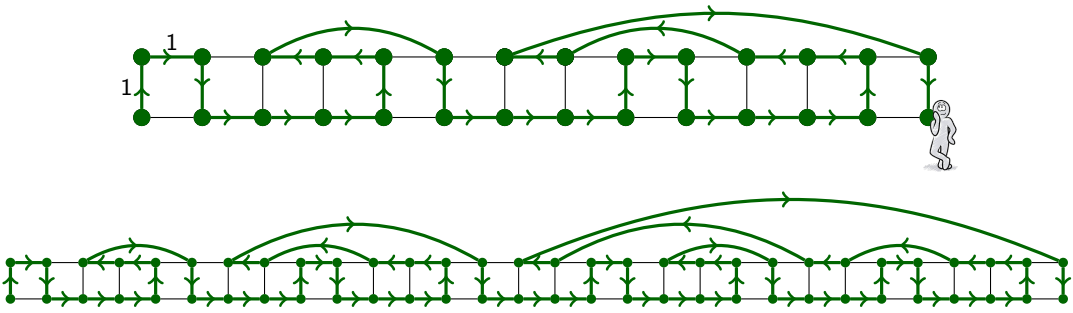
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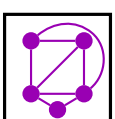
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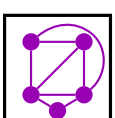
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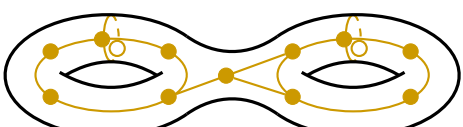
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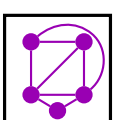
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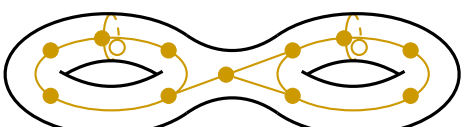
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- in this talk: generalization to minor-free graphs

Minor-free graphs

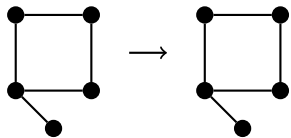
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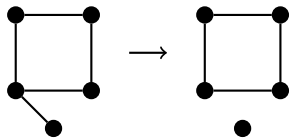
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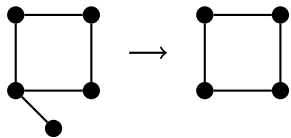
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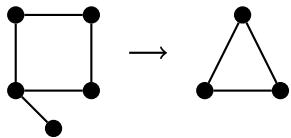
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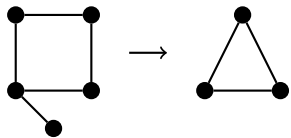
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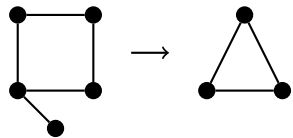
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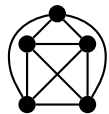
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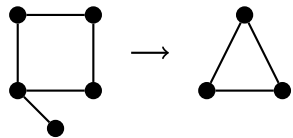
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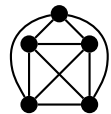
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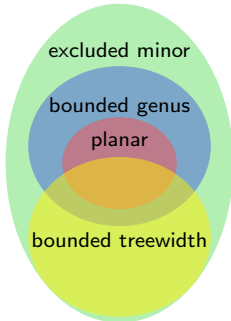
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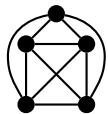
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Main result

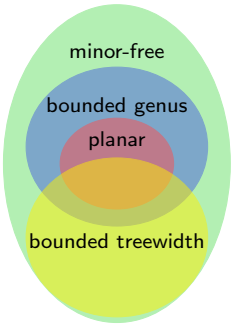
Theorem. For every graph H,
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(B., Disser, Heinrich, Schweitzer, 2023)

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The algorithm BLOCKING$_\delta$

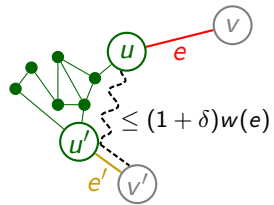
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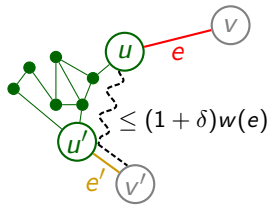
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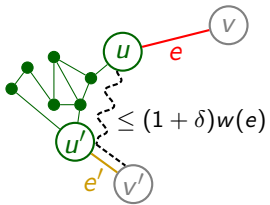
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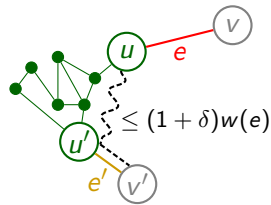


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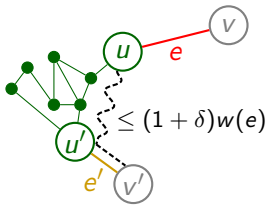
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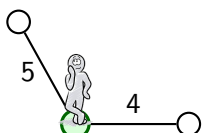
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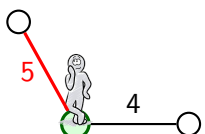


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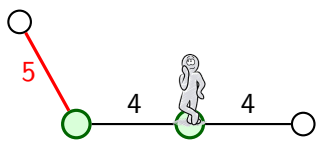


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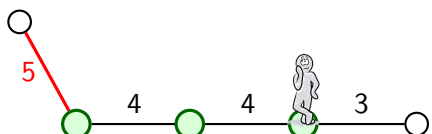


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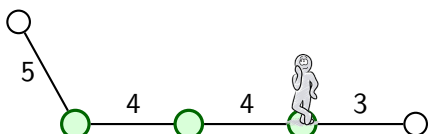


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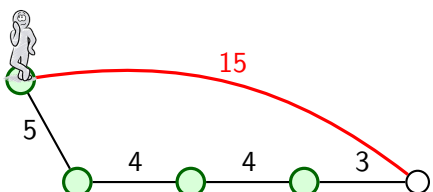


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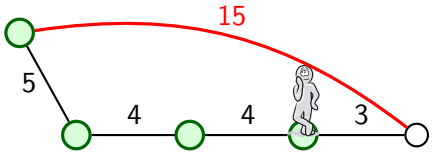


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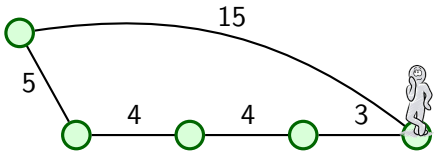


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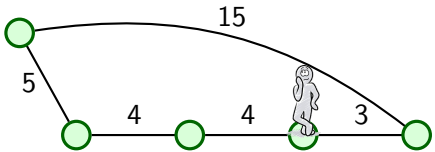


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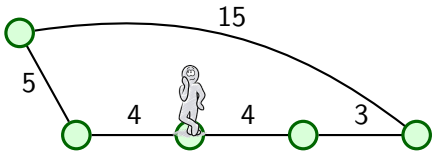


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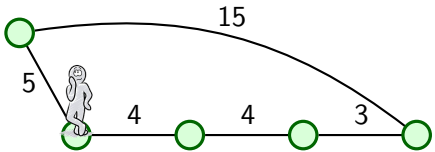


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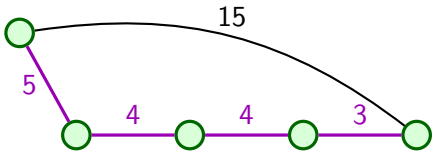


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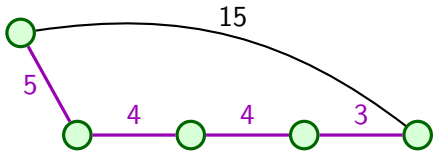
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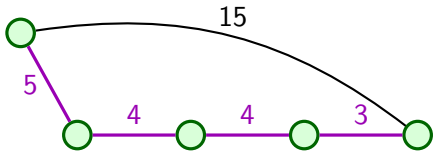
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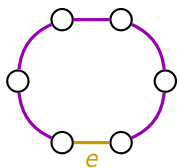
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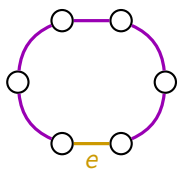


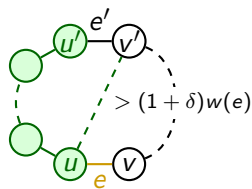
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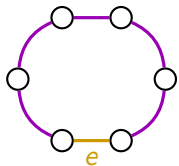


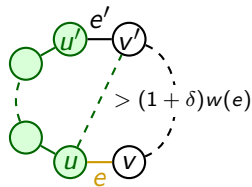
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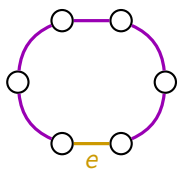
Definition. A subgraph $H \subseteq G$ is a $(1+\varepsilon)$-**spanner** of G if $\text{dist}_H(u,v) \leq (1+\varepsilon) \cdot \text{dist}_G(u,v)$ for all $u, v \in G$.

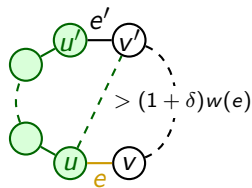
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Lemma. There is an MST of G such that every cycle C in $B \cup \mathrm{MST}$ fulfills $w(C \setminus \{e\}) > (1+\delta)w(e)$ for every $e \in C$.





Definition. A subgraph $H \subseteq G$ is a $(1+\varepsilon)$-**spanner** of G if $\mathrm{dist}_H(u,v) \leq (1+\varepsilon) \cdot \mathrm{dist}_G(u,v)$ for all $u, v \in G$.

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Goal. $\textsc{Blocking}_\delta$ is $O(1)$-competitive on minor-free graphs.

i.e.: $\textsc{Blocking}_\delta(G, v) \leq O(1) \cdot \textsc{Opt}(G)$ $\forall$ minor-free G and vertices v

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- Conjecture. Every graph has a spanner of stretch $2s-1$ and lightness $O(n^{1/s})$ (for every $s > 1$).
- Theorem. For every $\varepsilon > 0$ and fixed graph H, every H-minor-free graph has a $(1+\varepsilon)$-spanner of lightness $O(1)$.

(Borradaile, Le, Wulff-Nilsen, 2017)

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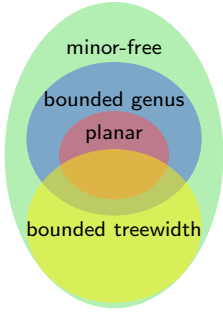
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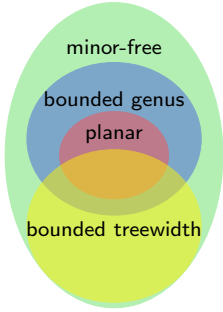
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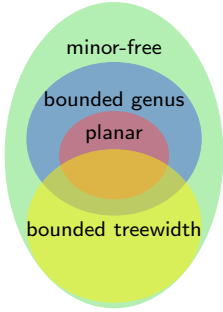
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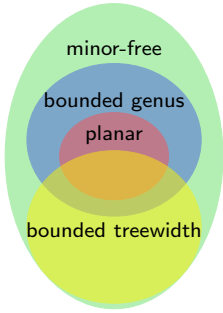
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